The 40th Anniversary of Toregas, Swain, ReVelle, and Bergman

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Preamble Notes: not a slide

- Some of you, or perhaps the majority are wondering….what is this all about?
- I remember well a special address by Jaque Thisse, maybe about ten years ago, to this group concerning a body of literature beginning with august roots like that of Hotelling and Weber. His intent, I suspect, was to bring everyone on board to a particularly interesting area of Regional Science, that of location and competition. He did a great job…..and I hope that I can do at least as half as good as he did, and if I do I will consider myself successful.
- Rather than talk about some of my current work, I have chosen, instead, to look back and discuss one of the important elements of Location Science, which was first presented to Regional Science some forty years ago.
- One of the impetuses for this is that at the San Francisco meeting of 2009, I heard someone ask a question of a presenter…"where is the significance test?"…as if all work in Regional Science was based upon some form of statistical analysis. This made me think that perhaps many of us should attempt to better communicate in a broad manner, so that we have a better appreciation for the field as a whole.
- I do not and cannot attempt to do as well as those like Laksmannan, Britton Harris, Jaque Thisse, David Boyce and others in dealing with the entire field of regional science.
- So, I have picked a topic that started out as being very pragmatic, an attempt to solve a real problem of the day.
- Now, with this as a preamble, let me turn to the task at hand.
- First, I lied…there were three papers

The three fundamental papers

- Toregas, Swain, ReVelle and Bergman, "The location of emergency service facilities" *Operations Research* (1971)
- Toregas and ReVelle, "Optimal location under time or distance constraints," presented at the 1971 RSA meetings at Ann Arbor, MI (published in *Papers of the Regional Science Association* (1972).
- Toregas and ReVelle (1973) "Binary logic to a class of location problems," *Geographical Analysis* ; 145-155.

Also, I need to fess up about the error on NARSC web site

Conference | Awards & Prizes | History & Officers | Regional Organizations

Charles ReVelle

At the 1972 North American Meetings, Professor Charles ReVelle began his involvement in Regional Science with a presentation with Constantine Toregas on facility location under time or distance constraints. This paper was soon published in Papers of the Regional Science Association. That paper along with companion papers in Geographical Analysis and Operations Research have been cited more than 300 times. Since that time, ReVelle has contributed many papers to the regional science literature, primarily involving location models for public and private sector application. In 1974, Professor ReVelle published a paper with Richard Church on the maximal covering location problem that is the second highest cited paper in the history of Regional Science. In the early 1980's ReVelle with his student John Current and a c

shortest path covering mode

found application in transit s d

Cohon paper in Regional Science and Urban Economics broke new ground in modeling the acquisition of land. That

Statement on web site associated with Chuck ReVelle being awarded the Walter Isard Award for Scholarly Achievement

UCSB

Wednesday, Nov. 9, 2011

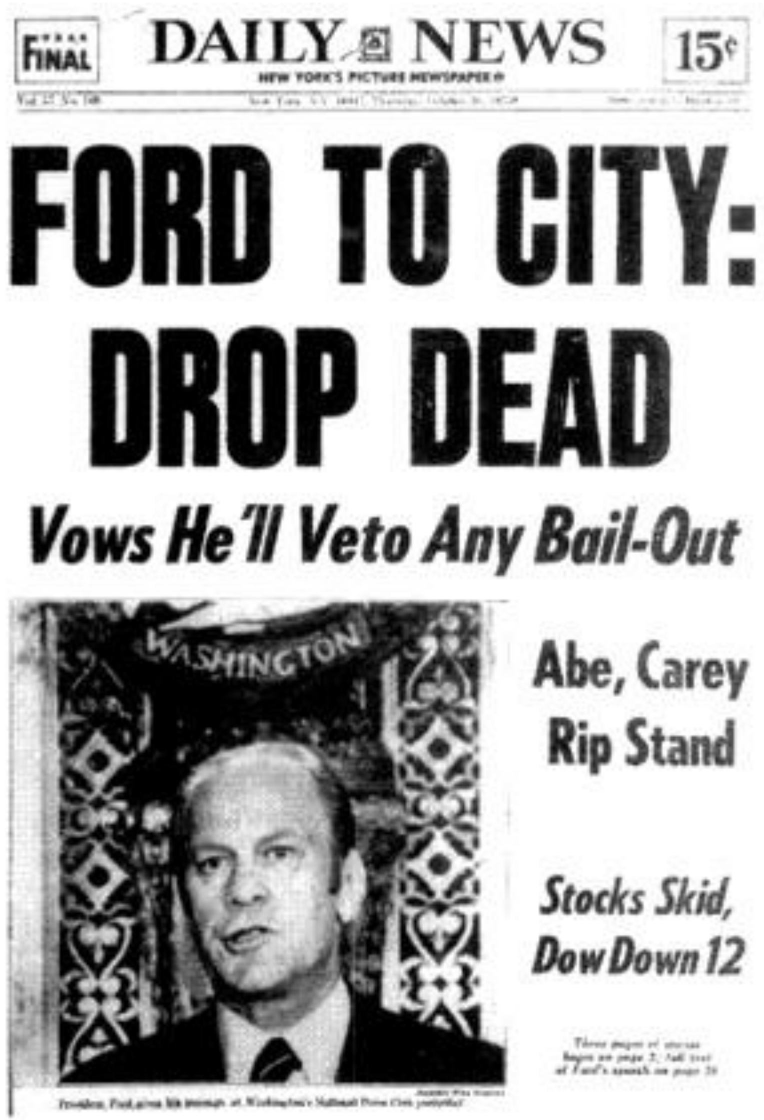
Outline of presentation

- I lied………
- The setting….let's go back to 1971 & the 70's in general
- The problem of Toregas……
- The significance of this work….
- Since Toregas……..
- Concluding thoughts………

Let’s go back to 1971

- Harvard tuition was $4,070
- Median family income was $9,870
- Richard Milhous Nixon was president
- Public Health Cigarette Smoking Act required a warning on cigarette packages starting 1971
- Occupational Safety and Health Act was passed
- Most popular TV show: “All in the Family”
- RSA meetings: Ann Arbor, MI

Some cities were in crisis


FINAL

DAILY NEWS

NEW YORK'S PICTURE NEWSPAPER

15¢

FORD TO CITY: DROP DEAD

Vows He'll Veto Any Bail-Out

Abe, Carey Rip Stand

Stocks Skid, Dow Down 12

Oct. 29, 1975

Computational resources

- In 1970, IBM announced a family of machines with an enhanced instruction set, called System/370
- fast disk drives
  - disk drive model 3330-11: 400 MB for $111,600 (1973) or $279/MB.
- CPU RAM prices exceeded $75,000/MB (you were lucky that you had access to a machine with 1 MB

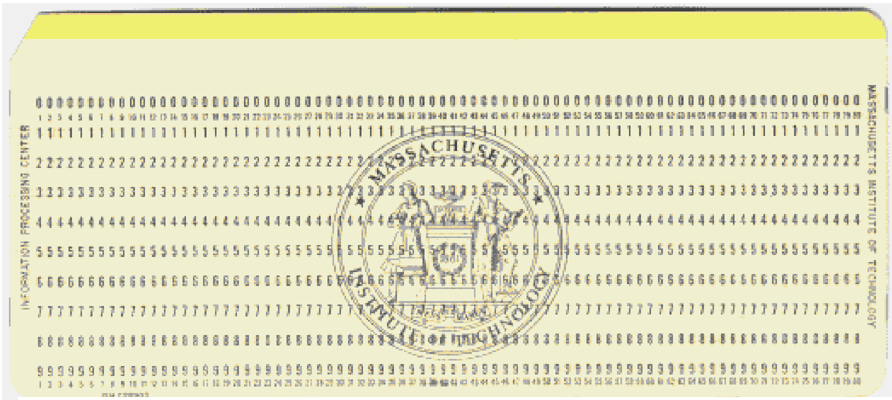
Coding

- Assembly language, Fortran, Cobol, Algol, PL/1

- No C, C++, objects, graphical user interface, no IDE

- No python, java, GIS, LINGO, Mosel, OPL, AMPL, MATLAB, R, GeoDa, CPLEX, ARC/GIS, MapInfo……….

Computer communications



Input media: punched cards

Output media: "green bar"

Solving Linear Programming Problems

- 1947, a team of 9 people solved Stigler's diet problem of 9 constraints and 77 variables using electronic calculators (took 120 man-days of effort)
- In the early 1960's Orchard-Hays wrote LP90/94 for the IBM 7094 system that could handle up to 1,024 constraints
- In the late 1960's IBM introduced MPS/360 which could handle 32,000 constraints
- P&G ran tests and found that it cost $70 to solve a 464 constraint, 1,084 variable problem with 90 binary variables using MPS/370 in 1977.

One last attempt at spinning a perspective……

D E Boyce, A Farhi, & R Weischedel (1973) "Optimal network problem: a branch and bound algorithm," *Environment and Planning* 5: 519-533.

This paper describes a budget constrained network design problem and solution algorithm …….computational experiments on several 10 node, 45 link problems…..times..3 to 400 seconds on an IBM 360/75 machine……(U. Penn.)

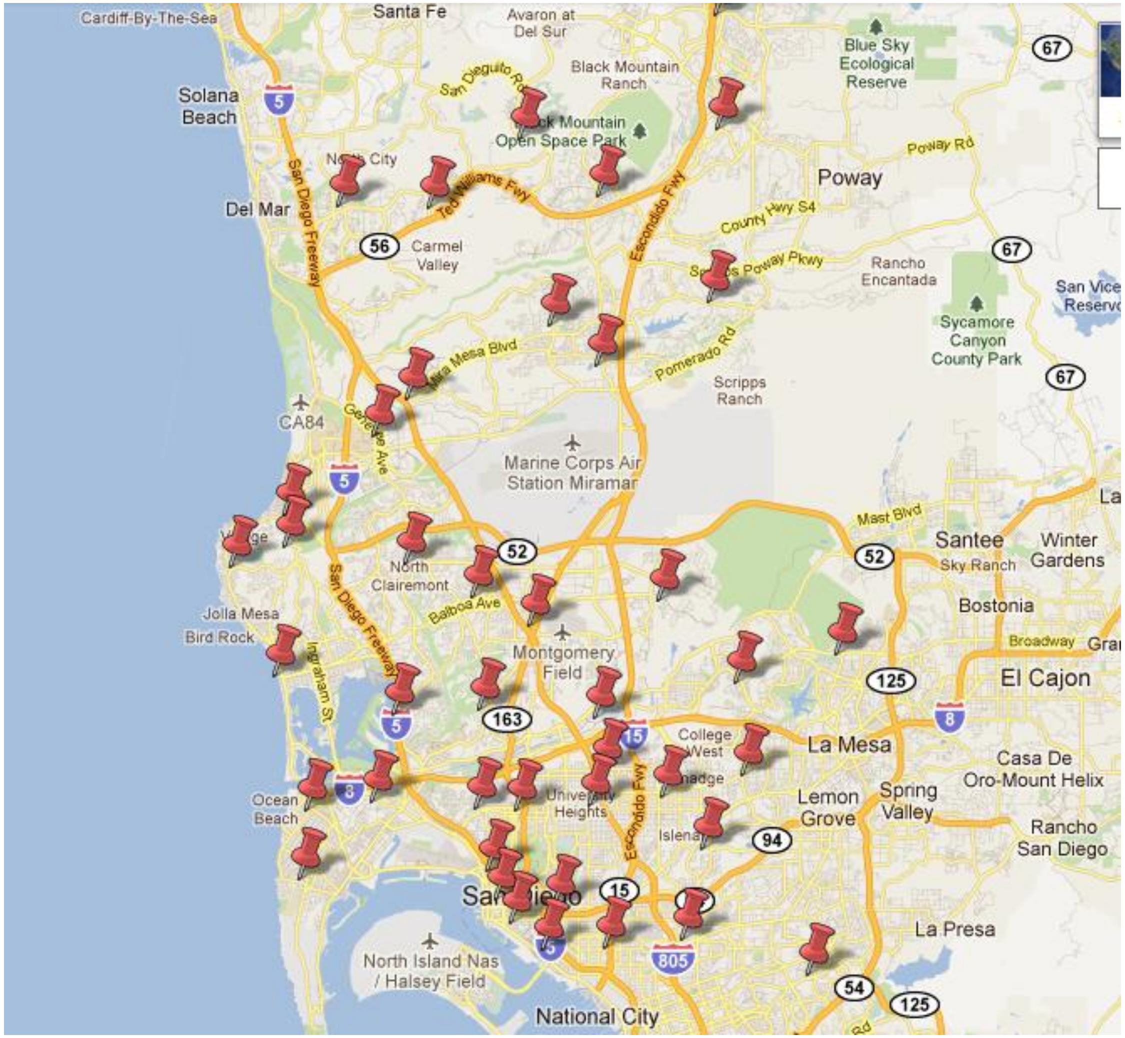
Perspective….the bottom line

- In a recession, with the DOW dropping 45% in the first few years of the decade
- City budgets in the toilet, NYC close to default
  - A need to make services more efficient
- Computational resources…… very limited

Now,…… what about Toregas, Swain, ReVelle and Bergman

……a focus on the efficient provision of emergency services….

Big business



Toregas & ReVelle, 1971 RSA presentation….

"For private sector location models, the objective is rather obvious, it is motivated by the objective of profit."

"In the public sector, the locational problem is a much more difficult one. There is no overriding objective in this field, and a variety of responses may be given to the simple question on the "best" location configuration of some service"

T&R continued……

“the difficulty in defining a direct measure of social utility leads to a search for some surrogate measure of public utility which may in some way approximate, the way in which an entire public may react to a proposed configuration”

“Consider the regional location of emergency service facilities, such as fire houses, ambulance depots, and police stations. Achieving coverage within a maximal distance or time is one of the major requirements which must be satisfied by an allocation scheme.”

The problem

- T&R quote from H.C. Huntley who stated (1970) for EMS:
  - "If a 15 minute response time is demanded, how many ambulances are required and where must they be positioned to provide reasonable assurance that this criterion is met"
- T&R argued that the concept of a maximal service distance or time may be utilized as a prime factor in the location of public service facilities & as a surrogate measure in providing public safety services

Setting Standards,…………………. an example of fire protection

For Fire: *distance based*

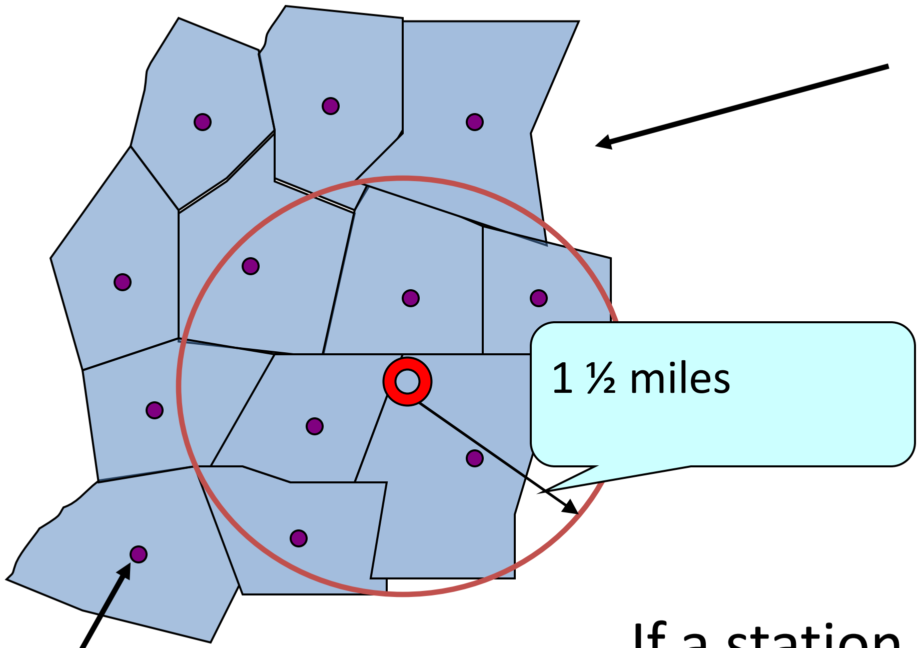
- Stations should be located so that neighborhoods are within a mile and a half from a station in a suburban area and one mile in dense urban areas

Formulating the problem

LSCP: find the minimum number and their locations for a set of fire stations, such that all neighborhoods are covered within the standard

LSCP- location set covering problem Toregas & ReVelle, 1971   RSA meetings Ann Arbor, MI

Depicting the problem geographically….

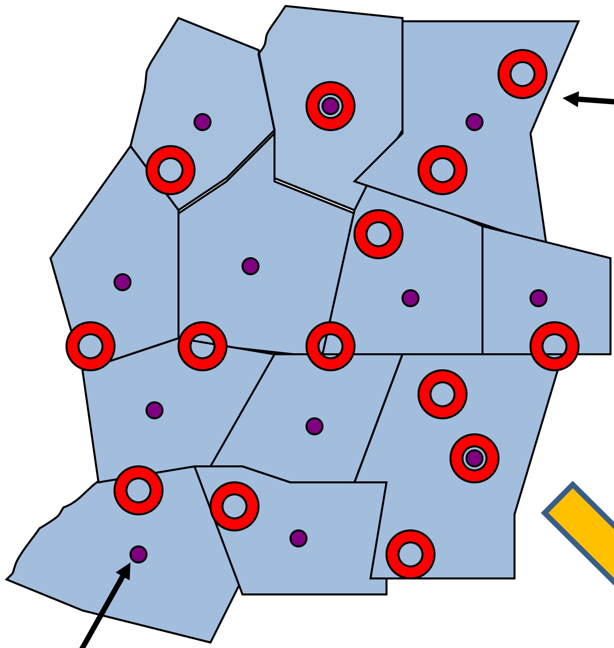


City divided into fire demand zones, zones are small

If a station covers a zone centroid, then it can adequately serve that zone

Each demand zone is represented by a centroid

Depicting the problem geographically….



Potential fire station locations are identified and depicted as red donuts

Each demand zone is represented by a centroid

Determine for each possible station location, the fire response zones that it can cover

Let's represent this is a formal way:

$$i = \text{an index to represent zones}$$

$$j = \text{an index to represent potential station sites}$$

$$a_{ij} = \begin{cases} 1, & \text{if a station at } j \text{ covers zone } i \\ 0, & \text{otherwise} \end{cases}$$

Sites

Zero-one coverage matrix

$a_{ij} =$

	1	**2**	**3**	**4**	**5**
1	1	1	0	0	0
2	1	1	0	1	0
3	0	1	1	1	0
4	0	0	1	1	1
5	0	0	1	0	1

Note: this information can be found in Toregas & ReVelle, 1972, Papers of thee RSA

What if we locate stations at sites 1 & 3?

Sites

$a_{ij} =$

	1	**2**	**3**	**4**	**5**
1	1	1	0	0	0
2	1	1	0	1	0
3	0	1	1	1	0
4	0	0	1	1	1
5	0	0	1	0	1

Notice that each demand is covered by at least one the two selected sites

Let's represent our site choices by decision variables

$$x_j = \begin{cases} 1, & \text{if site } j \text{ is selected for a station} \\ 0, & \text{otherwise} \end{cases}$$

For example, $x_1 = 1,\ x_2 = 0,\ x_3 = 1,\ x_4 = 0,\ x_5 = 0$

Site selection decisions

$a_{ij} =$

	x_1	x_2	x_3	x_4	x_5
	1	**2**	**3**	**4**	**5**
1	1	1	0	0	0
2	1	1	0	1	0
3	0	1	1	1	0
4	0	0	1	1	1
5	0	0	1	0	1

We've defined....

$$x_j = \begin{cases} 1, \text{ if site } j \text{ is selected for a station} \\ 0, \text{ otherwise} \end{cases}$$

Let's focus on a demand, in terms of our decision variables

For example, demand 1:

$$x_1 \quad + \quad x_2 \geq 1$$

........for demand 3:

$$x_2 \quad + \quad x_3 \quad + \quad x_4 \geq 1$$

$a_{ij} =$

	x_1	x_2	x_3	x_4	x_5
	1	**2**	**3**	**4**	**5**
1	1	1	0	0	0
2	1	1	0	1	0
3	0	1	1	1	0
4	0	0	1	1	1
5	0	0	1	0	1

We've defined....

$$x_j = \begin{cases} 1, \text{ if site } j \text{ is selected for a station} \\ 0, \text{ otherwise} \end{cases}$$

To cover each zone, we must meet all of the following conditions:

$$x_1 + x_2 \geq 1$$

$$x_1 + x_2 + x_4 \geq 1$$

$$x_2 + x_3 + x_4 \geq 1$$

$$x_3 + x_4 + x_5 \geq 1$$

$$x_3 + x_5 \geq 1$$

$a_{ij} =$

	1	2	3	4	5
1	1	1	0	0	0
2	1	1	0	1	0
3	0	1	1	1	0
4	0	0	1	1	1
5	0	0	1	0	1

We've defined....

$$x_j = \begin{cases} 1, \text{ if site } j \text{ is selected for a station} \\ 0, \text{ otherwise} \end{cases}$$

Now we can add the objective, which is to minimize the number of stations that we need:

$$\textit{Minimize} \quad z = x_1 + x_2 + x_3 + x_4 + x_5$$

subject to:

$$\begin{aligned} x_1 + x_2 &\geq 1 \\ x_1 + x_2 + x_4 &\geq 1 \\ x_2 + x_3 + x_4 &\geq 1 \\ x_3 + x_4 + x_5 &\geq 1 \\ x_3 + x_5 &\geq 1 \end{aligned}$$

The LSCP Model

In coverage set notation this is depicted as:

$$\text{Minimize } Z = \sum_{j} x_j$$

$$s.t.$$

$$\sum_{j \in N_i} x_j \geq 1 \quad \text{for each demand } i$$

$$x_j = 0,1 \quad \text{for each site } j$$

This model and computational testing using LP/IP can be found in Toregas, Swain, ReVelle, and Bergman (1971)

By solving the LSCP model for a range of *S* values, we can determine the following tradeoff:

Find the smallest maximal service distance for which you could afford deployment…..

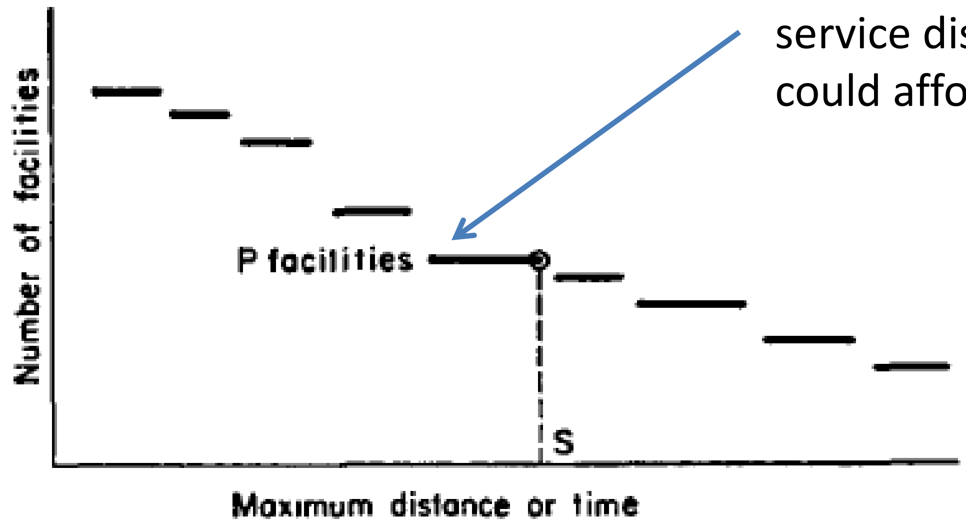


FIG. 4. Relationship Between Number of Facilities Located and Maximum Distance or Time Constraint.

Toregas & ReVelle, Binary Logic Solutions to a Class of Location Problems, *Geographic Analysis*, 1973.

Let's be real!

- We have a model, but computational resources are lacking…………
- The tradeoff curve is great for analysis, but we might have to spend the equivalent of a years' wage to generate it……
- Thus, the significance of the RSA presentation of Toregas & ReVelle in 1971

Reduction Rules

- "The present (Papers, RSA, 1972) work makes use of reduction techniques which were first introduced in the field of electrical engineering in the 1950's. The rules are best characterized by saying that they eliminate redundancy in the constraints and decision variables."
- "They have as well direct geographical interpretations in the context of the location problem."

Coverage Matrix

		Site				
		A	**B**	**C**	**D**	**E**
	1	1	1	0	0	0
Demand	**2**	1	1	0	1	0
	3	0	1	1	1	0
	4	0	0	1	1	1
	5	0	0	1	0	1

The presence of a one in the 0-1 coverage matrix means that Site *X* can cover demand *i*.

Definition-- Dominated Row

- **Dominated Row**-a row is dominated by another row if for every entry in the domineering row there is a corresponding entry in the dominated row
- *Motivating Idea...* if coverage is provided to a demand represented by a dominating row, then coverage is automatically provided to the dominated one

Dominated Row

		Site				
		A	**B**	**C**	**D**	**E**
Demand	**1**	1	1	0	0	0
	2	1	1	0	1	0
	3	0	1	1	1	0
	4	0	0	1	1	1
	5	0	0	1	0	1

Dominated Row

		Site					
		A	B	C	D	E	
Demand	1	1	1	0	0	0	domineering
	2	~~1~~	~~1~~	~~0~~	~~1~~	~~0~~	dominated
	3	0	1	1	1	0	
	4	0	0	1	1	1	
	5	0	0	1	0	1	

Definition-- Dominated Column

- **Dominated Column-** a column is dominated by another column if the other column has exactly the same entries as the dominated column (plus possibly even more entries)
- *Motivating Idea...* if a site B can cover the same demand as site A can, plus possibly other demand as well, site A is inferior to B in terms of coverage and should never be chosen.

Dominated Column

		Site				
		A	**B**	**C**	**D**	**E**
Demand	**1**	1	1	0	0	0
	2	1	1	0	1	0
	3	0	1	1	1	0
	4	0	0	1	1	1
	5	0	0	1	0	1

Dominated Column

		Site				
		A	**B**	**C**	**D**	**E**
Demand	**1**	1	1	0	0	0
	2	1	1	0	1	0
	3	0	1	1	1	0
	4	0	0	1	1	1
	5	0	0	1	0	1

dominated domineering

Definition-- Essential Column

- **Essential Column-** a column is essential if it is necessary to provide coverage to a demand
- *Motivating Idea*... There's no way out, site X must be chosen in order to cover a specific demand

Essential Column
(different matrix than previous example)

		Site				
		A	**B**	**C**	**D**	**E**
	1	1	0	1	1	1
Demand	**2**	1	0	1	0	0
	3	1	1	1	1	1
→	**4**	0	0	0	1	0
	5	0	0	1	1	1

Essential Column

(different matrix than previous example)

				Site				
				A	**B**	**C**	**D**	**E**
			1	1	0	1	1	1
Demand			**2**	1	0	1	0	0
			3	1	1	1	1	1
		→	**4**	0	0	0	1	0
			5	0	0	1	1	1

Essential Column

(different matrix than previous example)

		Site				
		A	**B**	**C**	**D**	**E**
Demand	**1**	1	0	1	1	1
	2	1	0	1	0	0
	3	1	1	1	1	1
→	**4**	0	0	0	1	0
	5	0	0	1	1	1

Must choose site D, because it's the only site that can cover demand 4

Reductions Algorithm for the LSCP

1. Eliminate All Dominated Rows
2. Eliminate All Dominated Columns
3. Site at all Essential Columns & eliminate rows covered by Essential Columns
4. Repeat steps 1,2,3 until no matrix is left or until no changes occur in one complete cycle of steps 1,2,& 3.

Solution (Step 1)

				Site				
				A	**B**	**C**	**D**	**E**
			1	1	1	0	0	0
Demand			**2**	1	1	0	1	0
			3	0	1	1	1	0
			4	0	0	1	1	1
			5	0	0	1	0	1

Step 1: Eliminate all dominated rows

Solution (Step 1)

				Site				
				A	**B**	**C**	**D**	**E**
		→	**1**	1	1	0	0	0
Demand			**2**	~~1~~	~~1~~	~~0~~	~~1~~	~~0~~
			3	0	1	1	1	0
			4	0	0	1	1	1
			5	0	0	1	0	1

Step 1: Eliminate all dominated rows

Row 1 dominates Row 2, remove Row 2

Solution (Step 1)

		Site				
		A	**B**	**C**	**D**	**E**
Demand	**1**	1	1	0	0	0
	3	0	1	1	1	0
	4	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~
	⇨ **5**	0	0	1	0	1

Step 1: Eliminate all dominated rows

Row 5 dominates Row 4, remove Row 4

Solution (Step 1)

				Site				
				A	**B**	**C**	**D**	**E**
			1	1	1	0	0	0
Demand								
			3	0	1	1	1	0
			5	0	0	1	0	1

Step 1: Eliminate all dominated rows

Solution (Step 1)

				Site				
				A	**B**	**C**	**D**	**E**
			1	1	1	0	0	0
Demand								
			3	0	1	1	1	0
			5	0	0	1	0	1

Step 1: Eliminate all dominated rows

Step 1 is done, no other rows are dominated by other rows

Solution (Step 2)

		Site				
		A	B	C	D	E
	1	1	1	0	0	0
Demand						
	3	0	1	1	1	0
	5	0	0	1	0	1

Step 2: Eliminate all dominated columns

Column B dominates Column A, remove Column A

Solution (Step 2)

				Site				
					B	C	D	E
			1		1	0	0	0
Demand								
			3		1	1	1	0
			5		0	1	0	1

Step 2: Eliminate all dominated columns

Columns D and E are dominated by C,
remove Columns D and E

Solution (Step 2)

				Site				
					B	**C**		
			1		1	0		
Demand								
			3		1	1		
			5		0	1		

Step 2: Eliminate all dominated columns

Solution (Step 2)

				Site				
					B	**C**		
			1		1	0		
Demand								
			3		1	1		
			5		0	1		

Step 2: Eliminate all dominated columns

Step 2 is done, no columns are dominated by other columns

Solution (Step 3)

		Site				
			B	C		
	1		1	0		
Demand						
	3		1	1		
	5		0	1		

Step 3: Site at "essential columns," and eliminate rows covered by those columns

Solution (Step 3)

				Site			
				B	C		
			1	1	0		
Demand							
			3	1	1		
			5	0	1		

Step 3: Site at "essential columns," and eliminate rows covered by those columns

Must choose Site B in order to cover Demand 1

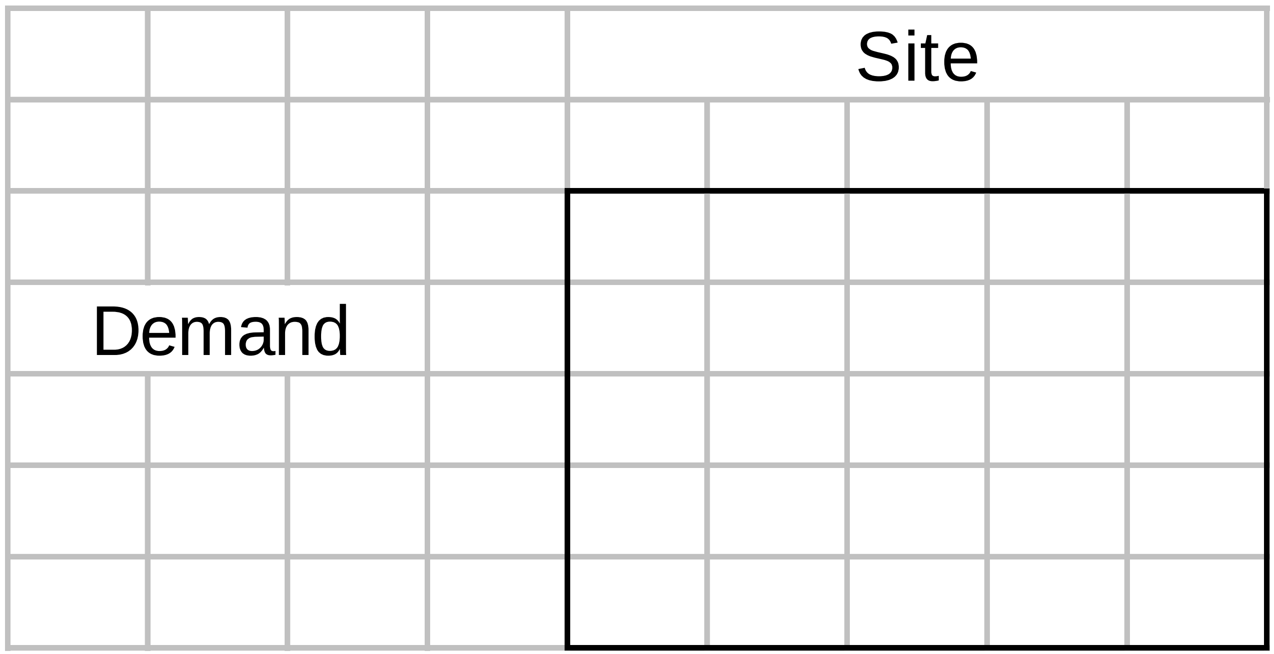
Solution (Step 3)

Demand \ Site			C		
5			1		

Step 3: Site at "essential columns," and eliminate rows covered by those columns

Must choose Site C in order to cover Demand 5

Solution (Final)



No matrix is left, so we're done!

Remember, we chose Sites B and C.

Coverage Matrix

				Site				
				A	B	C	D	E
			1	1	1	0	0	0
Demand			2	1	1	0	1	0
			3	0	1	1	1	0
			4	0	0	1	1	1
			5	0	0	1	0	1

Notice that we covered all demands with a minimum number of sites, namely B and C.

What if we end up with…

		Site		
		K	Q	T
Demand	**17**	1	1	0
	45	0	1	1
	51	1	0	1

This matrix can not be reduced any further……..it is a cyclic matrix

Cyclic matrices tend to be small, and can often be solved by inspection!

What this meant

- Reasonably sized LSCP problems were solvable using specialized code, followed by inspection of any remaining cyclic matrix
- Sometimes LP with a strong cut was necessary for resolving the cyclic matrix
- The bottom line: solving the LSCP was within reach of application by cities and counties

Toregas' accomplishments

- Toregas joined PTI, Inc. after receiving his PhD at Cornel University and later became its president/CEO
- PTI specialized in technology transfer to cities
- In 1977, ReVelle et al. in a *Health Services Research* Paper stated that nearly 100 cities in the US had asked PTI for assistance in using the LSCP model for fire service reorganization & planning.

There are many possible uses of "covering"

- Selecting Manpower for inspection teams
- Selecting Media for advertising
- Locating specialized hazardous waste response equipment
- Dynamic repositioning of equipment during high fire demands
- Locating bus stops along transit routes
- Locating EMS posting positions
- Selecting reserve sites for biological preservation
- Locating cruisers in the gulf of Aden to protect ships
- Locating surveillance cameras using viewsheds
- Locating guard posts and look out towers
- Analyzing settlement patterns (Aztecs & Egyptians)
- …….

uses, continued

- area wide warning sirens
- tow truck placement, Athens Olympic games
- radar systems for ports and rivers
- rain gauge network design
- emergency shelters
- health clinics
- apparel trim patterns on fabric
- optimal fortification
- air quality monitoring networks
- color tab design for false teeth
- …….

LSCP, a branch of location science

The classic problems of Location Science

- The p-median problem (a generalized network version of Weber)
- The simple plant location problem
- The capacitated plant location problem
- Covering problems:
  - The Location Set Covering Problem
  - The Maximal Covering Location Problem

The "seed" of LSCP, begat

Maximal covering location problem (Church and ReVelle)

Multi-objective max cover (Schilling, Cohon, ReVelle)

The fixed charge maximal cover problem (Church and Davis)

The maximal expected coverage problem (Daskin)

The partial covering problem (White & Case)

The "seed" of LSCP

Covering shortest path problem (Current, Cohon & ReVelle) inherent in transit design

The maximal covering path problem (Current, et al.)

The maximal availability location problem (MALP) (Hogan and ReVelle) handles equipment busyness

Facility Location and Equipment Emplacement (FLEET, MOFLO, TEAM & MOFLEET) ReVelle, Schilling, Elzinga & Church; Bianchi and Church)

The “seed” of LSCP

The planar location set covering & maximal covering problems:

- Points (Church)
- Area (Current and O’Kelly, Murray and O’Kelly; and many others)

Multiple cover models (BACOP, HOSC, multi-level LSCP) Daskin & Stern; Hogan & ReVelle, Church & Gerrard.

The “seed” of LSCP

Minimal impact covering (Murray et al.)

Threshold covering (Storbeck and Charnes)

Capacitated covering (Schilling and Pirkel)

Coherent covering (Sera and ReVelle)

Transit covering (Gleason; Murray, Stimson et al.)

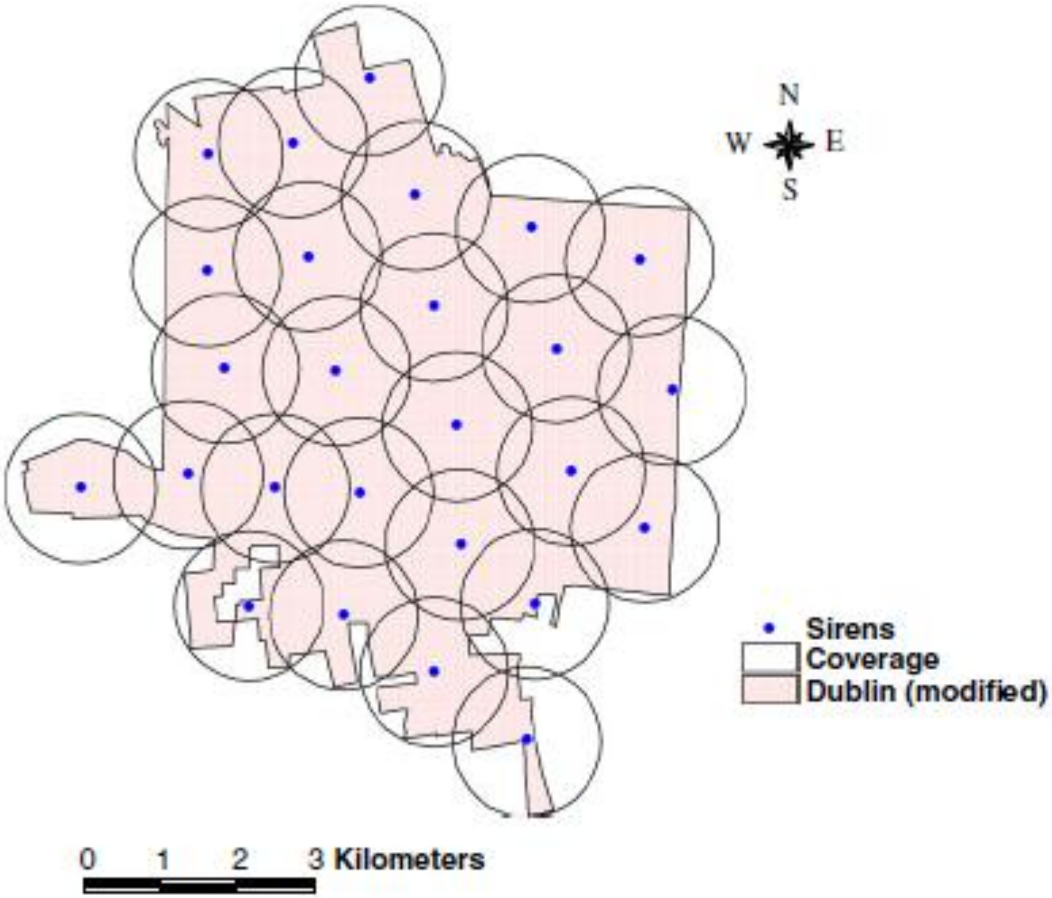
Weighted benefit and variable radius covering (Church and Roberts; Berman & Krass; Drezner)

Not to mention more recent work of Murray, Tong, Marianov, Eiselt, Batta, and many others

Time to stop

- The work of Toregas, et al. has had a profound impact on field of Location Science
- It represents a perfect example of technology transfer from a research university to public service agencies
- The citations alone from the 3 original papers by Toregas and the three highest cited articles on “max cover” exceed 2000 citations in web of science

Parting thought........



Wednesday, Nov. 9, 2011

Constantine Toregas